Investigating the Sensitivity of Frequency Response Function to Parameters in Equivalent Circuit of Transformer Winding Based on Two FRA Measurement Connection Ways

Song Wang [1], Shuhong Wang [1], *Senior, IEEE*, Hanke Feng[1], Ze Guo[1], Shuang Wang[1], Dongsheng Yuan[1], Hailin Li[1]

[1] State Key Laboratory of Electrical Insulation and Power Equipment, Faculty of Electrical Engineering, Xi'an Jiaotong University, Xi'an, 710049, China, shwang@mail.xjtu.edu.cn

As a reliable and non-destructive technique, frequency response analysis (FRA) has been widely recognized for transformer winding deformation detection and assessment. In this paper, frequency response function (FRF) are obtained by two different FRA measurement connection ways, namely end to end voltage ratio and transfer voltage ratio measurements. At the same time, according to the two ways, sensitivity of the FRFs to electrical parameters of the equivalent circuit model of the transformer winding are investigated. Based on Tellegen theorem, adjoint network method is derived to calculate the values of the sensitivity of FRF to parameters. By comparing the results of two ways, the variation laws of the sensitivity from the two different connection ways are determined. Besides, this study will give us a more deep-lever and accuracy understanding how the variation of parameters influence on the FRA results.

***Index Terms*—sensitivity analysis, frequency response analysis, adjoint network, transformer winding**

I. Introduction

FRA method is a popular and effective technique to detect and monitor transformer winding deformations [1]. In FRA method, a low voltage sinusoidal sweep frequency signal is injected into one terminal of the winding, and the final measurement signal is received from another terminal. Normally, analysis of the test results relies on the comparison between previous results and present results. If there is no initial FRA trace, it is also feasible to making a comparison among phases in this transformer. Over the recent years, many types of research on FRA method have been done to enrich and extend this practical technique. In some relevant literature [2]-[3], they used different simulation models to get the FRA test trace. In [4]-[5], authors focus on finding out the best FRA terminal connection way, which has a much more sensitive to winding movements, such as axis movement and radial movement. However, no matter which simulation model or terminal connection way is selected to analyze FRA test results, these achievements do not establish a quantitative relationship between the FRF and electrical parameters in the equivalent circuit. In this paper, firstly, by adjoint network method, the sensitivity of FRF to electrical parameters based on two different terminal connection ways are obtained to directly illustrate the impacts of parameters variation on FRA results. Secondly, the variation laws of the sensitivity from different connection ways are determined by comparison.

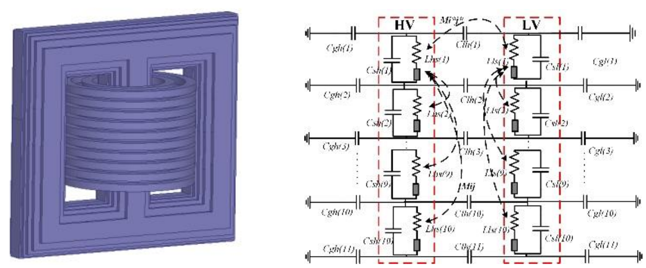
II. Transformer Model and Two Different Measurement Connection Ways

In this paper, as shown in Fig. 1(a), a single phase two-winding transformer model is employed to investigate the sensitivity of FRF to electrical parameters. The equivalent circuit of the two-winding transformer is shown in Fig. 1(b). Each disc of HV and LV winding is represented as a lumped element unit.

According to the energy balance method, the inductances and resistances of winding can be obtained in quasi-static approximation [6]:

$$L_i = 2\,\mathrm{Im}[\mathrm{j}\hat{W}_{ii}]/I_i^2, \quad R_i = 2\omega\,\mathrm{Re}[\mathrm{j}\hat{W}_{ii}]/I_i^2 \tag{1}$$

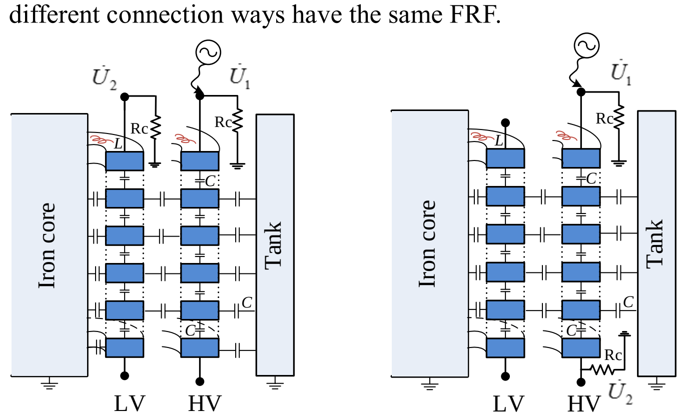
$$M_{ij} = [\mathrm{Im}[\mathrm{j}\hat{W}_{ii} - 0.5L_iI_i^2 - 0.5L_jI_j^2]]/\left(I_iI_j\right) \tag{2}$$



(a) (b)

Fig.1. (a) Two-winding transformer model.
(b) Lumped element equivalent circuit of the transformer model.

The test connection ways are shown in Fig. 2. The impedance of the measurement cable is 50 ohms (*Rc*). The input and out voltage signals are measured cross the *Rc*. Obviously, the two different connection ways have the same FRF.



(a) (b)

Fig.2. (a) Transfer voltage ratio connection way (TF).
(b) End to end voltage ratio connection way (EE).

The same FRF is expressed as:

$$|\boldsymbol{H}(\omega)| = |\dot{U}_2 / \dot{U}_1| \tag{3}$$

Since $H(\omega)$ has nothing to do with the value of input voltage, so input voltage can be set as 1V. In this case, we can get (4).

$$\begin{cases} |\boldsymbol{H}(\omega)| = |\dot{U}_2|, \text{ when } \dot{U}_1 = 1\text{V} \\ \Delta|\boldsymbol{H}(\omega)| = \Delta|\dot{U}_2| \end{cases} \tag{4}$$

III. Sensitivity of FRF to Electrical Parameters

A. *Definition of FRF Sensitivity to Parameters*

Variations of electrical parameters inevitably have an effect on the FRF. So, by sensitivity calculation approach, it is easy to set up the relationship between FRF and parameters. Mathematically, the sensitivity is the first-order derivative of model outputs with respect to the model parameters. The equation of FRF absolute sensitivity to the parameters is expressed:

$$D_{P_i}^H = \frac{\partial \boldsymbol{H}}{\partial P_i} \tag{5}$$

where, P_i is the ith (i=1,2,…,n) parameter in the circuit. Normalized sensitivity can be also got as:

$$S_{P_i}^H = \frac{\partial \boldsymbol{H}}{\partial P_i} \cdot \frac{P_i}{|\boldsymbol{H}|} \tag{6}$$

B. *Using Adjoint Network Method to Calculate Sensitivity*

Adjoint network method, which is derived from Tellegen theorem, is one of the most useful ways to get the network sensitivity. To a two-port linear network N, its adjoint network $\hat{N}$ has the same topological structure, as shown in Fig. 3. Symbol "^" represents the adjoint network. According to the Tellegen theorem [7], the fundamental sensitivity calculation equation based on the adjoint network can be derived as:

$$\sum_{k=1}^{m} (\hat{\boldsymbol{I}}_k \Delta \boldsymbol{U}_k - \hat{\boldsymbol{U}}_k \Delta \boldsymbol{I}_k) = 0 \tag{7}$$

where, k is the branch number and m is the sum of branches. $\boldsymbol{U}_k$ and $\boldsymbol{I}_k$ denote branch voltage and branch current vector of branch k, respectively. $\Delta\boldsymbol{U}_k$ and $\Delta\boldsymbol{I}_k$ are the variations of branch voltage and branch current vector, respectively, which are caused by a small perturbation of the relative parameter. The expansion of (7) can be written as:

$$\hat{\boldsymbol{I}}_1 \Delta \boldsymbol{U}_1 - \hat{\boldsymbol{U}}_1 \Delta \boldsymbol{I}_1 + \hat{\boldsymbol{I}}_2 \Delta \boldsymbol{U}_2 - \hat{\boldsymbol{U}}_2 \Delta \boldsymbol{I}_2 = -[\sum_R (\hat{\boldsymbol{I}}_{Ri} \Delta \boldsymbol{U}_{Ri} - \boldsymbol{U}_{Ri} \Delta \dot{\boldsymbol{I}}_{Ri}) \tag{8}$$
$$+ \sum_L (\hat{\boldsymbol{I}}_{Li} \Delta \boldsymbol{U}_{Li} - \hat{\boldsymbol{U}}_{Li} \Delta \boldsymbol{I}_{Li}) + \sum_C (\hat{\boldsymbol{I}}_{Ci} \Delta \boldsymbol{U}_{Ci} - \hat{\boldsymbol{U}}_{Ci} \Delta \boldsymbol{I}_{Ci})]$$

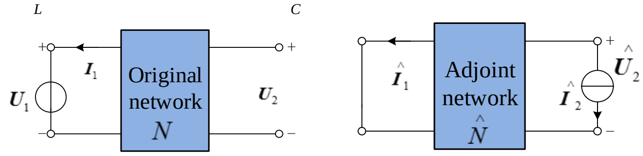


Fig.3. (a) original network and (b) adjoint network.

Assume that R_i has a little change, then the R_i turns into $R_i+\Delta R_i$. According to (8), the final form of the sensitivity of FRF to resistance R_i is got as:

$$\frac{\partial \boldsymbol{H}}{\partial R_i} = \frac{\partial \boldsymbol{U}_2}{\partial R_i} = -\boldsymbol{I}_{Ri} \hat{\boldsymbol{I}}_{Ri} \tag{9}$$

In a similar manner, we can get all expressions of the normalized sensitivity of FRF to electrical parameters, as listed in (10).

$$S_R^H = -\boldsymbol{I}_R \hat{\boldsymbol{I}}_R \frac{R}{|\boldsymbol{U}_2|}, \quad S_L^H = -\mathrm{j}\omega \boldsymbol{I}_L \hat{\boldsymbol{I}}_L \frac{L}{|\boldsymbol{U}_2|}, \quad S_C^H = \mathrm{j}\omega \boldsymbol{U}_C \hat{\boldsymbol{U}}_C \frac{C}{|\boldsymbol{U}_2|} \tag{10}$$

IV. Results of Sensitivity Calculation

To get more information about FRF sensitivity to parameters under different frequencies, the simulation frequency domain is set from 50Hz to 1MHz. Using (10), it is very convenient to get the sensitivity of FRF to the parameters. The results of *Csh* in TF way are shown in Figure. 4. Although sensitivity of FRF to each *Csh* has a little different, the change trend is similar. FRF sensitivity to *Csh*s are very small in the low frequency, but they increase with the increasing of frequency. Finally, in the high frequency, they tends to be a constant and the values of sensitivity decrease along with the serial number of *Csh* increase.

By the way, the other parameters results of sensitivity and the comparison results of the different ways will be detailedly presented in the full paper.

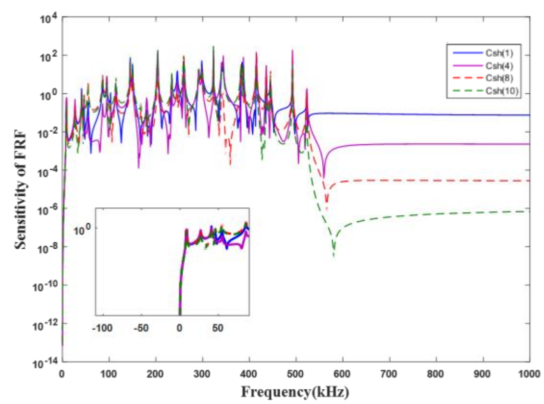


Fig. 4. Sensitivity of FRF to *Csh*s in TF test configuration.

References

[1] S. A. Ryder, "Diagnosing transformer faults using frequency response analysis," *IEEE Elect. Insul. Mag*., vol. 19, no. 2, pp. 16–22, Apr. 2003.

[2] A. Shintermirov, W. H.Tang, and Q. H. Wu, "A hybrid winding model of disc-type power transformers for frequency response analysis," Power Delivery, IEEE Transactions on, vol. 24, pp. 730-739, 2009.

[3] L. Guishu, S. Haifeng, Z. Xile, and C. Xiang, "Modeling of trans-former windings under very fast transient overvoltages," Electro-magnetic com-patibility, IEEE Transactions on, vol. 48, pp. 621-627, 2006.

[4] J. A. S. B. Jayasinghe, Z. D. Wang, P. N. Jarman, and A. W. Darwin, "Winding movement in power transformers: a comparison of FRA meas-urement connection methods," IEEE Trans. Dielectr. Elect. Insul., vol. 13, no. 6, pp. 1342–1349, Dec. 2006.

[5] S. O. Demokritov, "Brillouin light scattering spectroscopy of magnetic nanostructures," *IEEE Trans. Magn.,* submitted for publication.

[6] M. H. Samimi, A. A. S. Akmal, H. Mohseni, S. Tenbohlen, "Detection of transformer mechanical deformations by comparing different FRA con-nections," International Journal of Electrical Power & Energy Systems, Vol. 86, pp. 53-60, March. 2017.

[7] F. Yuan, A. Opal, "Sensitivity analysis of periodically switched linear cir-cuits using an adjoint network technique", Proceedings of the 1999 IEEE International Symposium on Circuits and Systems, ISCAS '99, vol. 5, pp. 331-334.